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On the Movement of the Solar System in Space.

By G. B. Airy, Esq., Astronomer Royal.

The object of the author in this memoir is to develope a new method for the determination of the direction and amount of the movement of the solar system in space, from a conviction of the inadequacy and defects of the methods hitherto in use. The first of these methods is that of Sir William Herschel, proposed by him when the number of known proper motions was very small. It consisted mainly in drawing upon a celestial globe a great circle representing the direction of proper motion of each star, and finding the point at which any two such circles intersect each other, which point, supposing the apparent proper motion of each star to arise entirely from a real motion of the sun, would be that to or from which the sun's motion is directed. This method was most happily adapted to the limited facts of observation then known, and the whole investigation considered one of the best specimens of Sir W. Herschel's enterprise and sagacity; but it is evidently insufficient for the discussion of proper motions of a great number of stars.

The second method, which is applicable to the discussion of the proper motions of larger catalogues of stars, has been in use up to the present time, and is of the following nature. A point is assumed as a probable approximation to the apex of solar motion; the direction of the arc of great circle drawn from the star's place to this point is ascertained as also the direction of the star's apparent proper motion; and the angle between these two directions multiplied by a certain factor is considered as the small residual errors in ordinary problems of combinations of observations; and all the equations thus formed are to be treated by the method of least squares.

In the author's opinion there are many objections to this method, of which the following are the most important:— First, that it requires the point to be approximately known, since an error of its assumed position equal in amount to the smallest which could be probably assigned to Sir W. Herschel's determination causes great confusion in the change of direction of the great circle drawn from the star's place to the assumed point of direction of solar motion, when the star is not very distant from that point; and, secondly, that the proper motions of some stars are so nearly the opposite to those implied in the assumption of the point, that a small change in the place of the point will change the angle of errors *per saltum* from $+179^\circ$ to -179°.

On such grounds as these the author considers the method employed very imperfect, and proposes one totally different, of which the following sketch will give a tolerably correct idea:—

Instead of using the apparent angular motions of the sun and stars, as exhibited on the surface of a globe, their linear

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movements are treated of by reference to rectangular co-ordinates, and the following advantages result from the adoption of this method. It is quite independent and complete, and does not require the assumption of a point determined by preceding investigations; and it gives exactly due weight to each observation subject to our antecedent judgment as to the general weight-multiplier to be attached to any class of stars defined by brilliancy or any other characteristic, with the exception of magnitude of proper motion.

In applying the method, the author takes for the origin of co-ordinates the centre of the sun at a fixed epoch, and for the plane of $x\,y$, the plane passing through the sun parallel to the earth's equator, the axis of x being drawn towards the first point of *Aries*, and that of y to the point whose right ascension is 90°; z is drawn parallel to the earth's axis and is positive towards the north.

Taking the sun's linear motion in a year or other definite time, and resolving it in the direction of x, y, z, it is not difficult to express the R.A. and N.P.D. of a star at the end of the year in terms of the initial values of R.A. and N.P.D., and of the sun's resolved motion in x, y, and z, multiplied into certain functions of R.A. and N.P.D. easily calculated. In fact, for a star at distance r, the expressions for its apparent R.A. and N.P.D., at the end of the year, will be respectively,

$$\text{Initial R.A.} - \frac{\sin \text{R.A.}}{r\,.\sin \text{N.P.D.}}\,\text{X} + \frac{\cos \text{R.A.}}{r\,.\sin \text{N.P.D.}}\,\text{Y},$$

and

$$\text{Initial N.P.D.} + \frac{\cos \text{R.A.}\cos \text{N.P.D.}}{r}\,\text{X} + \frac{\sin \text{R.A.}\cos \text{N.P.D.}}{r}\,\text{Y} - \frac{\sin \text{N.P.D.}}{r}\,\text{Z}$$

where $-$X, $-$Y, and $-$ Z, are the resolved parts of the yearly motion of the sun.

From which the change of star's apparent place produced by sun's real motion in direction of a parallel to the equator will be

$$-\frac{\sin \text{R.A.}}{r}\,\text{X} + \frac{\cos \text{R.A.}}{r}\,\text{Y}$$

and the change of apparent N.P.D. will be

$$\frac{\cos \text{R.A.}\cos \text{N.P.D.}}{r}\,\text{X} + \frac{\sin \text{R.A.}\cos \text{N.P.D.}}{r}\,\text{Y} - \frac{\sin \text{N.P.D.}}{r}\,\text{Z}.$$

The author then proceeds to show that where a number of equations given by different stars, the elements of linear motion of the different stars must be considered absolutely as chance-quantities to be treated in the same way as chance-errors of observation, subject, however, to one consideration

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(not treated of in this memoir) as to views which may be entertained respecting the probable inequalities of motion of the stars forming our nebular cluster, as measured in various directions. To this consideration he calls the attention of future inquirers, but as he has no means of forming a judgment on the nature of the motions of individual stars, he supposes that for every star the probabilities of movements of given magnitude in all directions are equal.

He next enters on the consideration of the effect of systematic errors which may have crept into the computations by which the proper motions have been determined. These proper motions are taken from Mr. Main's paper printed in vol. xix. of the *Memoirs*, and from another paper recently printed, and were deduced by comparison of the places of Bradley's stars as given in Bessel's *Fundamenta*, with the places given in the Greenwich Twelve-year and subsequent Six-year Catalogue, and therefore will be affected by any errors in Bradley's observations or Bessel's reductions. In addition, they will be affected by incorrect values of the constants of precession used in reducing the stars from Bradley's epoch to the modern Greenwich epochs. On the whole, they may consist, for right ascensions, in the error of equinox used in reducing Bradley's observations, together with errors of the constants of precession, and from north polar distances, in errors of the constants of precession, together with those arising from the defects of the Greenwich quadrants. To these systematic errors must be added the chance-errors of observation of each individual star not connected in any way whatever.

By comparison of the aggregates of the various terms before specified with the tabulated results of observations, equations of the following forms arise:—

$$-\frac{\sin \text{R.A.}}{r}\text{X} + \frac{\cos \text{R.A.}}{r}\text{Y} + m' \sin \text{N.P.D.} + n' \sin \text{R.A.} \cos \text{N.P.D.}$$

$$+\frac{\text{M}\xi}{r} + \text{Error in parallel} = \text{Proper Motion in parallel.}$$

and

$$\frac{\cos \text{R.A.} \cos \text{N.P.D.}}{r}\text{X} + \frac{\sin \text{R.A.} \cos \text{N.P.D.}}{r}\text{Y} - \frac{\sin \text{N.P.D.}}{r}\text{Z} + q -$$

$$n' \cos \text{R.A.} + \frac{\text{M}\eta}{r} + \text{Error in N.P.D.} = \text{Proper Motion in N.P.D.}$$

where $\frac{\text{M}\xi}{r}$ and $\frac{\text{M}\eta}{r}$ are the real apparent angular motions in the directions of parallel and meridian, m' and n' are the possible errors of m and n, the constants that enter into the formula of precession in R.A. and N.P.D., and q is the possible error of

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Bradley's observations in N.P.D., arising chiefly from the defects of the Greenwich quadrants.

Equations may now be formed on two extreme suppositions; first, that the irregularities of Proper Motion are entirely due to chance-error of observation; secondly, that they are entirely due to peculiar motions of the stars. The author is inclined to think that the second supposition corresponds most nearly with the truth, especially when the stars to which the calculation is applied are taken from a catalogue of large proper motions; he, however, proceeds to show how the final equations for determination of X, Y, and Z, must be found, according to the rules of the Theory of Probabilities, on both these extreme suppositions.

For the values of *r*, Mr. Airy has been guided by the assumptions of Struve (*Introd. in Cat. Nov. Stell. Dup.*), the stars being divided into seven classes from the 1st magnitude to the 8th.

As it would have been a work of enormous labour to make use of all the stars in Mr. Main's catalogues, amounting to about 1200, those only have been used which have proper motions above a certain limit assigned by the author of the catalogues, but without any distinct motives of preference.

The author here makes a remark which is of great importance in the determination of the solar motion, and which it is best to express without abridgement in his own words:

"It appears to me that the propriety or impropriety of retaining the stars whose proper motions are extremely large will depend entirely upon our assumption as to the practical reference for zero of space. In void space, there can be no zero either of linear ordinate or of angle. Any person who endeavours to form a notion of such zero will find that his mind sinks under the effort. We are compelled therefore to adopt, as our zero of space, the place of one body, or the mean of the places of many bodies. And, to whatever extent we adopt a number of bodies in forming our mean, to that extent our deductions of all kinds will apply. Thus, if we include in our treatment the stars with very large proper motions (61 *Cygni*, *μ Cassiopeiæ*, &c.), we may expect to obtain the movement of the solar system, as referred to the mean of all the stars of which we treat, including 61 *Cygni*, *μ Cassiopeiæ*, &c.; if we exclude them, we obtain the movement as referred to the mean of all the stars except those. The retention or exclusion of those stars appears then to be quite arbitrary, provided that we give the proper interpretation to our final results. In the investigations of this paper, I retain these stars, so far as they are included in Mr. Main's catalogue."

The author then proceeds to the solution of the equations on the two suppositions before-mentioned, and obtains the following results; R being the sun's whole proper motion expressed

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in seconds of arc as seen from a star of the first magnitude, and A and D the right-ascension and north declination respectively of the point to which that motion is directed.

First supposition.

$$\begin{aligned} R &= 1{\cdot}269 + 0{\cdot}245 \times n' - 1{\cdot}690 \times q \\ A &= 256^\circ\,54' - 1{\cdot}393 \times n' \\ &= 256^\circ\,54' - 79^\circ\,82' \times n' \\ D &= 39^\circ\,29' - 0{\cdot}159 \times n' - 1{\cdot}616 \times q \\ &= 39^\circ\,29' - 9^\circ\,12' \times n' + 92^\circ{\cdot}61 \times q \end{aligned}$$

Second supposition.

$$\begin{aligned} R &= 1{\cdot}912 + 0{\cdot}472 \times n' - 2{\cdot}190 \times q \\ A &= 261^\circ\,29' - 1{\cdot}998 \times n' \\ &= 261^\circ\,29' - 114^\circ\,47' \times n' \\ D &= 24^\circ\,44' - 0{\cdot}114 \times n' - 2{\cdot}486 \times q \\ &= 24^\circ\,44' - 6^\circ\,51' \times n' - 142^\circ{\cdot}43 \times q \end{aligned}$$

The author considers the results of the second assumption to be more worthy of confidence than those of the first, and proceeds to give the numerical evidence of the probability that the motion of the solar system is not very different from that given by it, by forming the sums of the squares of the observed proper motions in parallel and in N.P.D., and then forming the sums of the squares of the residual errors of the proper motions after the application of the terms depending on the solar motion. The results according to the second supposition are,

Sum of squares of Motion in Parallel	Uncorrected	54·4450
	Corrected	48·4217
Sum of squares of Motion in N.P.D.	Uncorrected	48·1206
	Corrected	45·3129

"From these the reader may form an opinion on the legitimacy of the general conclusions. I do not believe that the fundamental suppositions are sufficiently certain to justify any investigation as to the probable range of error in the assigned position of the line of the sun's motion.

"The most remarkable feature in the result is, however, the very great difference between the velocity of the solar motion as found from this investigation, namely, 1″·912, and that found by M. Otto Struve in his *Bestimmung der Constante der Præcession*, namely, 0″·339." This Mr. Airy attributes to the fact that the stars used in his own investigations are exclusively those having large proper motion, and enters largely into the explanation which depends, he considers, on the choice made in

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the apparent evidences of proximity of the stars, that is between brightness and magnitude of proper motion.

To the memoir is attached an appendix containing the elements of every sort used in the calculations, including the values of the functions of R.A. and N.P.D. which are necessary in the formation of the equations of condition.

Note on the Development of the Disturbing Function in the Lunar Theory. By Sir J. W. Lubbock, Bart.

Mr. Cayley, in a paper recently published, has pointed out some errors in the development of R given by me in my work on the Lunar Theory, and amongst others in the numerical coefficient of the term of which the argument is $2\tau + \xi - 3\xi'$, which I called $+\frac{45}{64}$, but which should be $-\frac{845}{64}$

$$\frac{dR}{de} = \frac{r\,dR}{dr}\,\frac{dr}{r\,de} + \frac{dR}{d\lambda}\,\frac{d\lambda}{de}$$

λ being the true longitude.

If we consider the term of which the argument is $2\tau + \xi$, in order to have the part multiplied by e, it is sufficient to take,

$$\frac{r\,dR}{dr} = 2R_1 \cos 2\tau \qquad \frac{dR}{d\lambda} = -2R_1 \sin 2\tau$$

$$\frac{dr}{r\,de} = -\cos\xi \qquad \frac{d\lambda}{de} = 2\sin\xi$$ see p. 26 Lunar Theory, Part 1.

$$\begin{aligned}\frac{dR}{de} &= 2R_1 \cos 2\tau \times -\cos\xi - 2R_1 \sin 2\tau \times 2\sin\xi \\ &= R_1\{-\cos(2\tau+\xi) - \cos(2\tau-\xi) \\ &\qquad + 2\cos(2\tau+\xi) - 2\cos(2\tau-\xi)\} \\ &= R_1\cos(2\tau+\xi) - 3R_1\cos(2\tau-\xi).\end{aligned}$$

When the term only involves the first power of e, as in this instance, $\int \frac{dR}{de}\,de = R$, or the coefficient of the term in R is the same as that of the corresponding term in $\frac{dR}{de}$.

The preceding result is entirely independent of ξ', and, moreover, would have been the same whatever multiple of ξ' had accompanied 2τ, so that as

the coefficient of cos $(2\tau+\xi)$ = the coefficient of cos 2τ
„ cos $(2\tau+\xi-\xi')$ = „ cos $(2\tau-\xi') = -\frac{21}{8}$
„ cos $(2\tau+\xi-2\xi')$ = „ cos $(2\tau-2\xi') = -\frac{51}{8}$
„ cos $(2\tau+\xi-3\xi')$ = „ cos $(2\tau-3\xi') = -\frac{845}{64}$